The International Towing Tank Conference (ITTC) is concerned with the qualitative execution of model scale tests and has such a long tradition in updating its quality manual by different technical committees. One of these committees is the Manoeuvring Committee, which received as a task of reference the update of the ITTC procedure Uncertainty Analysis for Manoeuvring Predictions based on Captive Manoeuvring Tests [1]. Until presently this guideline covers the following topics:

- The uncertainty induced by uncertainties in hull shape, planar motion geometry, rudder shape, control parameters, etc.
- The propagation of these uncertainties towards simulation predictions.

In the present procedure possible uncertainties induced by data reduction operations are not treated with detail. Moreover, according to [2] “the results of the PMM tests show a very large uncertainty. This is almost entirely due to the large Type-A uncertainty inherent in the measured force and moment data.” For this reason the 29th ITTC Manoeuvring Committee received as one of their terms of reference the study of the effect of data reduction techniques on the outcome of captive model tests for manoeuvring purposes. To have an idea of realistic noise levels, Figure 1 shows a typical, but raw
result of time series as measured on the benchmark hull KCS during a captive manoeuvring test program carried out in the frame of SIMMAN 2014.

Typically the sway force is measured at two different locations $x_F, x_A$ along the longitudinal axis of the ship which can then be combined to a sway force $Y$ and a yaw moment $N$:

$$ Y = Y_F + Y_A $$
$$ N = x_F Y_F + x_A Y_A $$

On a measured signal $F$, a Fourier analysis is commonly applied with three harmonics [2].

$$ F = F_0 + \sum_{j=1}^{3} \left[ F_{a_j} \cos(j\omega t) + F_{b_j} \sin(j\omega t) \right] $$

The Fourier coefficients $F_{a_j}$ and $F_{b_j}$ can then be used to compute the hydrodynamic derivatives, for instance, in case of linear manoeuvring model:

$$ Y_r v + (Y_r - m) \dot{v} + (Y_r - m u) r + Y_r \dot{r} = 0 $$
$$ N_r v + N_r \dot{v} + N_r r + (N_r - I_{zz}) \dot{r} = 0 $$

The following derivatives can be identified during a harmonic yaw test with $r = r_A \cos \omega t$:

$$ Y_r = m u + \frac{Y_{a_1}}{r_A} $$
$$ Y_r = m x_G - \frac{Y_{b_1}}{\omega r_A} $$
$$ N_r = m x_G u + \frac{N_{a_1}}{r_A} $$
$$ N_r = I_{zz} - \frac{N_{b_1}}{\omega r_A} $$

The basic research question is consequently how the noise affects the uncertainty on these derivatives. A second research question is how filtering, applied either in pre- or in postprocessing, affects these uncertainties. To this aim tests have been carried out at Akishima Laboratories (Japan), with and
without the use of analogue filtering. The present paper proposes a methodology to assess these uncertainties.

## 2 Uncertainties induced by noise in straight line tests

The present work is in fact an extension of the research carried out by Brouwer et al., reported in [4, 5] and currently adopted by the ITTC [6]. To set the ideas, the present method is here summarised. A time series $x_i(t)$ with finite length $T$ is considered as a sample record of an ergodic stationary random process. The sample average $m_i$ is an estimate of the true mean of the process $\mu_x$:

$$m_i = \frac{1}{T} \int_0^T x_i(t) \, dt$$

(10)

and the sample variance $s_i^2$ is an estimate of the true variance of the process $\sigma_x^2$:

$$s_i^2 = \frac{1}{T} \int_0^T [x_i(t) - m_i]^2 \, dt$$

(11)

According to [7], this can also be written as:

$$s_m = \sqrt{\frac{2}{T} \int_0^T \left[ 1 - \frac{\tau}{T} \right] C_{xx}(\tau) d\tau}$$

(12)

With:

- $s_m$: the standard deviation of the mean and also a measure of its uncertainty;
- $T$: the sampling interval or measuring length;
- $\tau$: the time difference or lag;
- $C_{xx}(\tau)$: the autocovariance function for a stationary process:

$$C_{xx}(\tau) = \frac{1}{T} \int_0^T x_i(t) \cdot x_i(t + \tau) \, dt - \mu_x^2$$

(13)

For a stationary process, $s_m$ should decay with increasing sample length. According to Brouwer et al. [4], an approximation of the standard deviation, and thus the uncertainty of the mean, is given by

$$u_1 = \sqrt{\frac{1}{T} \int_0^T \left( 1 - \frac{\tau}{T} \right) C_{xx,biased}(\tau) d\tau}$$

(14)

Where $C_{xx,biased}(\tau)$ is the biased estimator for the autocovariance (to reduce numerical instability):

$$C_{xx,biased}(\tau) = \left( 1 - \frac{\lfloor\tau\rfloor}{T} \right) C_{xx}(\tau)$$

(15)

Equation (14) can consequently be used to determine the uncertainty on the mean of a time signal which is affected by external noise. This corresponds also to the uncertainty of $F_0$ given in equation (3), provided that the true harmonic oscillating signal is subtracted from the measured time series.

## 3 Experimental case

### 3.1 Captive harmonic yaw test

The extension of the case will be first examined using the time series measured in 2010 on the KCS while executing captive harmonic yaw tests in the frame of SIMMAN 2014. More information on the
ship model KCS (at scale 1/52.667) can be found in [8]. The tests have been executed in the Towing Tank for Manoeuvres in Confined Water at FHR (co-operation FHR, Ghent University). The reader is referred to [9] for more details on this tank. For here it is important to mention that the towing carriage is equipped with a full CPMC which allows to steer the three horizontal motion modes independently.

Nine repetitions of the captive harmonic test mentioned in Table 1 will be discussed. Mind that the repetitions were mixed with other test types, to assess the true repeatability.

<table>
<thead>
<tr>
<th>Table 1. Details of the captive harmonic yaw test carried out with the KCS at 20% ukc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Longitudinal ship velocity $u$</td>
</tr>
<tr>
<td>Drift angle $\beta$</td>
</tr>
<tr>
<td>Yaw amplitude $\psi_A$</td>
</tr>
<tr>
<td>Test frequency $\omega$</td>
</tr>
<tr>
<td>Calibration time</td>
</tr>
<tr>
<td>Acceleration time</td>
</tr>
<tr>
<td>Steady state time</td>
</tr>
<tr>
<td>Deceleration time</td>
</tr>
<tr>
<td>Sampling interval</td>
</tr>
</tbody>
</table>

For the present tests with the KCS, the dynamometers to measure the lateral forces were fitted at the following positions with respect to the ship’s bound coordinate system shown in Figure 2:

- $x_F = 0.988 \pm 0.002$
- $x_A = -1.025 \pm 0.004$
3.2 Frequency analysis

A first step in the analysis of the measured time series is the analysis of the occurring frequencies. Figure 3 shows a detailed result for the first captive harmonic yaw test that has been carried out. Clearly these results can be filtered with a 4 Hz or 3 Hz low pass filter, which is commonly applied prior to modelling. At the same time, parts of the noise are of course being filtered, which has also an effect on the assessment of the uncertainty (see 6). On the same graph both first and third order Fourier fits are shown as well.\(^1\)

If the 3\textsuperscript{rd} order Fourier fit is the true signal, the remainder of the signal has to be categorized as undesired noise, in the same fashion as the subtraction from the mean from a steady straight line test (see section 2). This remaining part has been plotted in Figure 4a, but one can clearly see that this

\(^1\) Most computations in this paper have been carried out using the open source library MathNet (MathNet.Filtering 0.4.0 and MathNet.Numerics 4.5.1).
signal is not stationary, even without applying the so-called TST as elaborated by Brouwer et al. [5], which means that the method to assess the uncertainty of the mean does not seem applicable here.

When analysing the results in the frequency domain some noise peaks occur near 0 Hz, 3 Hz and 5 Hz. These components have to be ascribed to the excursion of the PMM mechanism and are thus not coincidental.

3.3 Monte Carlo simulations

In order to be able to apply the method elaborated in section 2, a simpler problem is solved first. Multiple databases of 1,000 artificial test results have been created by adding a synthetic time series to the first order Fourier fit. Such synthetic time series consists of Gaussian white noise with zero mean. By definition this noise follows a Gaussian distribution. A random noise generator was used, which could be controlled by setting the desired amplitudes or spread.

Each of the 1,000 artificial tests is then subjected to a Fourier analysis as described in 3.2. This can be regarded as a Monte Carlo simulation, with the true result being the original Fourier curve and with uncertainty the variance observed over the package of 1,000 artificial tests. The mean and the standard deviation of each harmonic component over the 1,000 tests are summarized in Table 2.

<table>
<thead>
<tr>
<th>Case Component</th>
<th>True result (sway force fore)</th>
<th>Gaussian noise with 1 N spread</th>
<th>Gaussian noise with 7.5 N spread</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Deviation</td>
<td>Mean</td>
</tr>
<tr>
<td>( F_0 )</td>
<td>-0.4807206</td>
<td>0.025677</td>
<td>-0.4802</td>
</tr>
<tr>
<td>( F_{a1} )</td>
<td>-3.107326</td>
<td>0.035997</td>
<td>-3.10709</td>
</tr>
<tr>
<td>( F_{b1} )</td>
<td>-9.202377</td>
<td>0.035176</td>
<td>-9.21046</td>
</tr>
<tr>
<td>( F_{a2} )</td>
<td>0</td>
<td>0.037003</td>
<td>0.001024</td>
</tr>
<tr>
<td>( F_{b2} )</td>
<td>0</td>
<td>0.0362</td>
<td>-0.00144</td>
</tr>
<tr>
<td>( F_{a3} )</td>
<td>0</td>
<td>0.035486</td>
<td>0.00643</td>
</tr>
<tr>
<td>( F_{b3} )</td>
<td>0</td>
<td>0.036599</td>
<td>-6.1E-05</td>
</tr>
</tbody>
</table>

Table 2 provides just some examples, as more variations have been simulated. Nevertheless some interesting trends can be observed:

![Figure 4 – Fourier analysis of the remaining noise signal, sway force fore.](image-url)
• in all cases the true result is predicted within mean ± deviation;
• in all cases the smallest uncertainty is observed for the constant term $F_0$ in the Fourier series;
• the harmonic terms $F_{a_i}$ and $F_{b_i}$ seem to have an equal uncertainty, but always larger than the constant term;
• the ratio between the harmonic and the constant terms is more or less 1.4.

This last empirical law can be confirmed with any base signal and any database size of synthetic signals (up to 10,000 artificial tests). This leads to the hypothesis that the uncertainty on the harmonics is constant and $\sqrt{2}$ times the uncertainty of the constant term. This hypothesis is elaborated in the next section.

Formula (14) was also applied to one artificial test and gave 0.02335 for 1 N spread and 0.16033 for 7.5 N spread, which are the same order of magnitude, but the Monte Carlo simulations give more conservative results, probably because of the biased estimator for the autocovariance.

4 Propagation of noise uncertainties in Fourier analysis

4.1 Theoretical evaluation

A short literature review has been conducted in order to assess the propagation of uncertainties through Fourier analysis, see for instance [10, 11, 12], however, the algorithms become quickly very complex and seem mostly applicable for higher frequency problems (acoustics, electronics). To the authors’ best knowledge the previously described empirical relationship has not been reported yet. However, in this section, this relationship is given a more solid basis, without pretending to act as a mathematical proof.

Assume a signal composed of $N$ samples $x_i + dx_i$ with $dx_i$ the component of a Gaussian noise distribution. The first harmonic of the signal is composed of:

$$a_1 = \frac{2}{N} \sum x_i \cos \omega t_i$$  
$$b_1 = \frac{2}{N} \sum x_i \sin \omega t_i$$

(16)  
(17)

The sensitivity to the uncertainties at position $i$ is equal to

$$c_i = \frac{\partial f}{\partial x_i} = \cos \omega t_i \text{ or } \sin \omega t_i$$

(18)

for $a_1$, respectively $b_1$. These uncertainties propagate through the summation. Because the noise follows a Gaussian distribution, each sample by itself is uncorrelated with the others, thus for instance the uncertainty on $a_1$ becomes (the same methodology can be followed for $b_1$):

$$da_1^2 = \left( \frac{2}{N} \right)^2 \sum c_i^2 \ dx_i^2$$

(19)

The term $\sum dx_i^2$ is a distribution of random variables (noise) with mean 0 and deviation $\sigma$, and as such the term is equal is equal to $\sigma^2 \chi^2_{N-1}$ (chi square distribution with $N - 1$ degrees of freedom), these random variables are however each multiplied with $c_i^2$. The average increase of the variances is:
\[
\frac{1}{N} \sum c_i^2 = \frac{1}{N} \sum \cos^2 \omega t_i = \frac{1}{N} \int \cos^2 N \, dN = \frac{1}{N} \left( \frac{N}{2} + \sin \left( \frac{2N}{4} \right) \right) \approx \frac{1}{2}
\]

for the cosine term and

\[
\frac{1}{N} \sum c_i^2 = \frac{1}{N} \sum \sin^2 \omega t_i = \frac{1}{N} \int \sin^2 N \, dN = \frac{1}{N} \left( \frac{N}{2} - \sin \left( \frac{2N}{4} \right) \right) \approx \frac{1}{2}
\]

for the sine term. The constant approximation is valid for a sufficient number of samples \(N\). In both cases the variance is increased with \(\frac{1}{2}\) for sufficiently large \(N\). In other words:

\[
d_a = \left( \frac{2}{\sqrt{N}} \right)^2 \sum c_i^2 dx_i^2 = \frac{4}{N} \left( \frac{1}{N} \sum c_i^2 \right) \sum dx_i^2 = \frac{4}{N^2} \sigma^2 \chi_{N-1}^2 = \frac{2}{N} \sigma^2 (N - 1) \approx 2 \sigma^2
\]

The uncertainty term on the harmonic is therefore: \(d a_1 = \sqrt{2} \sigma\), which supports the empirical relationship.

4.2 Application of this theory

Based on the above the uncertainty on every harmonic can be estimated as \(\sqrt{2}\) times the uncertainty of the constant term or the mean, provided the uncertainty of the latter is known. The uncertainty of the mean can be found with the following the method of Brouwer et al. [4,5] as explained in section 2. Once the uncertainty of the Fourier coefficients is estimated, its propagation can also be analysed through the derivatives (eventually with new Monte Carlo simulations).

5 Equivalent Gaussian noise

5.1 Monte Carlo simulations: part II

In the original stated problem (section 3.2), the noise of the experimental case is not Gaussian and the previously described theory is not strictly valid. The noise of the experimental case is rather proportional in a sense it increases with decreasing amplitude of the yawing table and thus increasing yaw rate \(r\) (see Figure 3). One could argue that the yaw rate dependent derivatives have possibly a larger uncertainty compared to the other derivatives.

In order to test the effect of proportional noise, new Monte Carlo simulations have been carried out using the same methodology as depicted in section 3.3, however this time 1,000 artificial tests have been created using the Fourier fit as true signal and supplemented with a noise spectrum as shown in Figure 4b. 1,000 spectral variations were obtained by adding a Gaussian variation on top to the spectrum.
The problem here is that the phases of each frequency component seem to follow a uniform distribution. This was also assumed to transform the results to the time domain, however, one can appreciate some shifts in maximal noise over the different artificial test results. The results of the Monte Carlo simulations have been summarised in Table 3.

<table>
<thead>
<tr>
<th>Case Component</th>
<th>True result (total sway force)</th>
<th>Noise distribution mock-up</th>
<th>Equivalent Gaussian</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_0$</td>
<td>0.356396179</td>
<td>0.352202</td>
<td>0.363692</td>
</tr>
<tr>
<td>$F_{a_1}$</td>
<td>-0.609353261</td>
<td>-0.6135</td>
<td>-0.61407</td>
</tr>
<tr>
<td>$F_{b_1}$</td>
<td>-4.243554116</td>
<td>-4.23319</td>
<td>-4.24272</td>
</tr>
<tr>
<td>$F_{a_2}$</td>
<td>0</td>
<td>0.002549</td>
<td>0.000253</td>
</tr>
<tr>
<td>$F_{b_2}$</td>
<td>0</td>
<td>-0.00983</td>
<td>0.001766</td>
</tr>
<tr>
<td>$F_{a_3}$</td>
<td>0</td>
<td>0.002402</td>
<td>0.001311</td>
</tr>
<tr>
<td>$F_{b_3}$</td>
<td>0</td>
<td>0.013217</td>
<td>0.005864</td>
</tr>
</tbody>
</table>

Again it seems that the relation between the harmonics and the average remain $\sqrt{2}$. However, for proportional noise, formula (14) is not applicable. A technique can be found based on seakeeping practice. The area of a wave spectrum is proportional to the variance of the sea state and any spectrum that has the same area in the frequency domain corresponds to a same amount of noise. The idea is then to build a Gaussian noise spectrum that has the same area as the measured noise spectrum. This noise spectrum is here referred to as Equivalent Gaussian noise.

The same methodology was followed and 1,000 artificial tests have been created with equivalent Gaussian noise. These results can also be found in Table 3. The ratio $\sqrt{2}$ for the uncertainties between the average and the harmonics is logically maintained, but unfortunately the average uncertainty is not exactly matched and provides rather an upper limit for the expected uncertainty.

5.2 Proposed algorithm

The following algorithm is proposed to determine the uncertainty of the different Fourier components:

- Perform a Fourier analysis on the raw signal;
- Subtract the Fourier fit from the raw signal to obtain the noise signal;
- Check whether this noise signal is stationary, for instance by applying the TST test from Brouwer et al. [5];
  - If this is the case, formula (14) can be applied to find the uncertainty of the mean.
  - If this is not the case Monte Carlo simulations should be performed. As an alternative hypothesis, the noise can be transformed to a Gaussian equivalent noise, which has the same spectral area. This Gaussian equivalent noise seems only capable to predict an upper limit of the uncertainty of the mean.
- The uncertainty of the mean has to be multiplied with $\sqrt{2}$ to obtain the uncertainties of the higher order harmonics.

This algorithm is open for discussion and should be investigated more thoroughly in the future. At least it seems sufficiently practical to be used in daily tank operations.

6 Effect of the use of analogue filters

6.1 Introduction

A common practice in many tanks is to directly apply an analogue filter when performing the tests. Only an ideal filter will be able to maintain the same information. However, here it is not the intention to discuss the uncertainty induced by realistic filters, the reader is referred to relevant literature for this topic, for instance [13, 14]. The question is rather how the results of the algorithm described in 5.2 are affected by the filtering method. From Table 2 it is already clear that a larger Gaussian noise amplitude will induce a larger uncertainty on the mean. Applying any filter will hence mathematically reduce that uncertainty.

6.2 Experimental test program

To investigate the described problem, a captive manoeuvring test program has been executed with the benchmark bare hull JBC [15] at Akishima Laboratories (Mitsui Zosen) (scale factor 101.81, deep water condition, $Fr = 0.1$). This program was executed twice: one without applying any analogue filters, and once with the application of an analogue 2nd order Bessel filter, of low pass type, which removes all noise above 1 Hz. The sampling frequency was in both cases 10 Hz. Figure 6 shows the results for the same harmonic yaw test in both programs.
6.3 Analysis of the noise

In the first place the noise in the test results have been analysed by subtracting the first order Fourier fit from the measurements. Figure 7 shows the noise characteristics both in the frequency and in the time domain. The filtered series show some dominant low frequent peaks which can be ascribed to fitting errors, whereas the raw result can be categorized as Gaussian, especially compared to the distribution shown in Figure 5.

Figure 6 – Harmonic yaw test with the JBC: $\omega = 0.398 \text{ rad/s}$, $\psi_A = 21.75^\circ$, $Fr = 0.1$, including 1 start-up cycle. Top: program without filter. Bottom: program with analogue low pass filter (1 Hz).

Figure 7 – Left: Fourier analysis of the noise signal from Fig. 6, effect of analogue filtering. Right: time series of noise signals from Fig. 6, including a 1 Hz LP software filter applied on the raw data.
One remarkable fact is that on the raw result a 1 Hz low pass filter has been applied (using a Hamming window), but that even then larger noise peaks appear compared to the analogue filter. The transfer function of the analogue filter has been analysed, see Figure 8, and revealed that the 2nd order Bessel filter has a characteristic which reduces its gain very gently even around the cut off frequency of 1 Hz. This may explain the fitting peaks in Figure 7, but also serves as a warning to take care of a good transfer function when filtering.

The noise signals shown in Figure 7 were also subjected to the TST developed by Brouw et al. [5]. The result is shown in Figure 9. The raw signal can be considered sufficiently steady, whereas the filtered signal shows a “hockey stick” behaviour, revealing unsteadiness. Although this example only considers a single test, it indicates that the raw signals can be examined with the algorithm developed in section 5.2. This has been done for all tests and all horizontal degrees of freedom. Table 4 shows the result for the test depicted in Figure 6. For this test, one can observe that the differences in test results cannot be attributed to noise uncertainty solely.

![Figure 8 – Transfer function of the applied analogue filter.](image)

![Figure 9 – Brouwer et al. TST test [5] for the noise signal from Fig. 7, effect of analogue filtering. Left: TST-A, right: TST-B](image)

### Table 4. Results of the noise uncertainty assessment of the test shown in Fig. 6.

<table>
<thead>
<tr>
<th>Case</th>
<th>Component</th>
<th>Raw result</th>
<th>Result with analogue filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>F₀</td>
<td>-0.149 ± 0.023</td>
<td>-0.002 ± 0.017</td>
<td>0.000 ± 0.004</td>
</tr>
<tr>
<td>F_{a₁}</td>
<td>0.003 ± 0.032</td>
<td>0.190 ± 0.024</td>
<td>0.746 ± 0.005</td>
</tr>
<tr>
<td>F_{b₁}</td>
<td>-0.030 ± 0.032</td>
<td>1.177 ± 0.024</td>
<td>0.769 ± 0.005</td>
</tr>
</tbody>
</table>
Moreover, the fact that a hockey stick is present for the given test, exaggerates the predicted noise for the results with analogue filter. In general the algorithm will predict an uncertainty which is proportional to the measured noise amplitudes. This can also be confirmed by comparing the Monte Carlo simulations for different spreads in Table 2. For white Gaussian noise, an analogue filter will imply a proportional reduction of these noise amplitudes and the proportion is dictated by the cut off ratio. For the present tests, executed at 10 Hz and filtered to 1 Hz, this means that the noise uncertainty will be reduced by a factor 10 when applying the algorithm described in section 5.2, in other words, the “real” noise uncertainty from a filtered signal will be the cut off ratio times larger than the computed value.

6.4 Propagation of the uncertainty to the hydrodynamic derivatives

To investigate the propagation of uncertainty towards the hydrodynamic derivatives, here focussed on the sway force derivatives, the results have been made non dimensional as follows:

\[
Y' = \frac{Y}{\frac{1}{2} \rho g L T u^2}
\]

\[
r' = \frac{r L}{u}
\]

\[
r' = \frac{r L^2}{u^2}
\]

The way the tests have been conducted at Akishima Laboratories \((r = r_A \sin \omega t)\), \(Y_{a1}\) should be used to determine \(Y_r\) and \(Y_{b1}\) to determine \(Y_r\). This has been done in Figure 10, but observe that in the current results the own inertial terms of the ship model are still included, however, this does not change anything to the discussion. The points based on the raw signal are fitted with their standard deviations based on the noise evaluation. Two black lines were drawn, which indicate the zone of possible results based on a 95% confidence interval. From the results it is clear that both the raw and filtered signal results fall within that zone and that statistically the deviations could be ascribed to noise. However, the points of the analogous filter (which according to the cut off ratio have probably the same uncertainty) are mostly located to the boundary, which indicates that probably other factors play a role in the differences of the result.

\[\text{Figure 10 – Determination of the derivatives, based on harmonic yaw test results.}\]

Especially \(Y_r\) seems affected by the noise. As already observed the noise levels remain constant over the different harmonics, and smaller values will therefore be more uncertain when the signal to noise
ratio is low. This is especially the case for higher order harmonics, which may as well be a result of spurious noise, see for instance $Y_{\text{err}}'$ in Figure 11.

![Graph showing determination of $Y_{\text{err}}'$ based on harmonic yaw with drift test results.]

Figure 11 – Determination of $Y_{\text{err}}'$, based on harmonic yaw with drift test results.

A final example is the effect of the noise measured during straight line tests at different drift angles, see Table 5. The slope of this relationship allows to determine the $Y_{\text{err}}'$ derivative. To assess the propagation uncertainty the method explained in Appendix F of [1] has been followed. $9^4$ Monte Carlo simulations have been carried out, covering 9 possible solutions in the 2% - 98% uncertainty range for each of the 4 drift angles. The following 95% probability ranges were found:

- For the raw data: $[-0.36148, -0.34714]$.
- For the filtered data, assuming the same uncertainties: $[-0.33518, -0.3208]$.

In this case there is no overlap between the intervals and the differences cannot be accounted for by noise only.

<table>
<thead>
<tr>
<th>Drift angle</th>
<th>Raw result [kgf]</th>
<th>Result with analogue filter [kgf]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-7.5^\circ$</td>
<td>$-0.2869 \pm 0.0074$</td>
<td>$-0.2778$</td>
</tr>
<tr>
<td>$-2.5^\circ$</td>
<td>$-0.0693 \pm 0.0046$</td>
<td>$-0.0754$</td>
</tr>
<tr>
<td>$2.5^\circ$</td>
<td>$0.0937 \pm 0.0056$</td>
<td>$0.0622$</td>
</tr>
<tr>
<td>$7.5^\circ$</td>
<td>$0.2882 \pm 0.0038$</td>
<td>$0.2584$</td>
</tr>
</tbody>
</table>

Table 5. Results of the noise uncertainty assessment for the sway force in straight line tests

7 Conclusions

In this article the uncertainties induced by noise and filtering have been discussed in the frame of analysis of the results of experimental captive manoeuvring tests. In the first place an extension is proposed to the method introduced by Brouwer et al. [5] to assess the uncertainties in case of PMM tests. By performing Monte Carlo simulations with artificial PMM test results an empirical relationship was found between the uncertainty of the mean or $0^\text{th}$ harmonic and the uncertainty of the higher harmonics. The uncertainties of the higher harmonics have all the same value and are $\sqrt{2}$ times larger than the uncertainty of the mean. This empirical relationship can also be supported by a theoretical evaluation in case of a Gaussian white noise distribution.
A steady noise distribution is also needed to be able to evaluate the uncertainty of the mean. The steadiness can be evaluated for instance with the TST method, also introduced by Brouwer et al. [5], however in some cases the noise is not steady, but rather proportional, for instance it increases or decreases with carriage excursions. In such case, the method seems difficult to apply, however, replacing the noise distribution by an equivalent white Gaussian one, meaning that they have the same spectral area, seems to give an upper limit for the uncertainty.

Because the uncertainties induced by noise depend on the amount of noise, applying filters can have a sheltering effect on these uncertainties. In general, for the determination of manoeuvring derivatives, it seems preferable to perform tests without analogue filtering, assess the noise uncertainties, and if needed apply a software filter on the measured raw data. If this is not feasible, it is recommended to multiply the noise uncertainties with the filter cut off ratio. As the noise seems to induce constant uncertainties in the higher harmonics, especially the non linear terms should be investigated for their significance.

The scientific community is invited to test the hypotheses put forward in this paper and if necessary further develop them.

8 Acknowledgements

Joris Brouwer and Jan Tukker from MARIN are acknowledged for providing additional explanations to their method.

9 References


